

MINIMAL SPINNING STRING

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Abstract

Minimal $N = 1/2$ supersymmetric extension of bosonic Polyakov's string are constructed. This model is natural generalization of Di Vecchia-Ravndal superparticle. The classical sector of the model are investigated, Noether currents and Virasoro supercondition are found. Minimal spinning string is more simple, than the standard $N = 1$ spinning string of Neveu-Schwarz-Ramond and has a number of unusual properties such as a chiral symmetry, parabolic type of equations of movement, non-triviality fermionic sectors for closed strings only and e.t.c.

Standard $N = 1$ supersymmetric extension bosonic strings admits, as is known, only two variants of further expansion – $N = 2$ and $N = 4$ [1]. Unexplored, however, there is the sector of the half-whole dimensions, the existence of which can be assumed, proceeding from existence of the appropriate theories of superparticles. A minimal superparticle of such type is $N = 1/2$ a superparticle Di Vecchia-Ravndal [2, 3]. The purpose of the given work is construction $N = 1/2$ of supersymmetric expansion bosonic strings. Such spin string is minimal string model, from which superparticle Di Vecchia-Ravndal can be received at aspiration of the size of a string to zero.

We shall consider in D -dimensional Minkowski space a world sheet, describing bosonic a string $x^\mu(\tau, \sigma)$, $\mu = 0, 1, 2, \dots, D-1$, where τ and σ – according to temporary and space parameter of points on sheet.

We shall generalize space of parameters $\sigma^a = \{\tau, \sigma\}$, $a = 1, 2$ up to superspace $B_L^{2,1} = \{\sigma^a, \theta\} = {}^0B_L \otimes {}^0B_L \otimes {}^1B_L$, where 0B_L , 1B_L – accordingly an even and odd subset of Grassmann algebra B_L [4]. On superspace $B_L^{2,1}$ there are the transformations of parameters, realizing the group SUSY:

$$\begin{array}{ll} \text{Translation } (T) : & \text{Supertransformation } (S) : \\ \left\{ \begin{array}{l} \sigma^a \rightarrow \sigma^a + \alpha^a \\ \theta \rightarrow \theta \end{array} \right. & \alpha \in {}^0B_L \quad \left\{ \begin{array}{l} \sigma^a \rightarrow \sigma^a + i\rho^a \varepsilon \theta \\ \theta \rightarrow \theta + \varepsilon \end{array} \right. \quad \varepsilon \in {}^1B_L . \end{array} \quad (1)$$

Here ρ^a – some numerical constants. At transformations T and S (1) is saved 1-form

$$\omega^a = d\sigma^a - i\rho^a \theta d\theta , \quad (2)$$

specifying metric in superspace

$$dS^2 = \eta_{ab} \omega^a \omega^b . \quad (3)$$

The metric tensor can be chosen in conformal-flat form $\eta_{ab} = \text{diag}(-1, 1)$ [1].

Thus on $B_L^{2,1}$ superexpansion of Minkowski metric is received

$$dS^2 = \eta_{ab}\omega^a\omega^b = g_{KL}d\Omega^K d\Omega^L ; \quad K, L = 0, 1, 2$$

$$\begin{pmatrix} -1 & 0 & i\rho^0\theta \\ 0 & -1 & -i\rho^1\theta \\ i\rho^0\theta & -i\rho^1\theta & 0 \end{pmatrix} \quad d\Omega^K = \{d\tau, d\sigma, d\theta\} \quad (4)$$

On superspace of parameters $B_L^{2,1}$ it is possible to set superfield

$$X^\mu : B_L^{2,1} \longrightarrow \bigotimes_{\mu=0}^{D-1} {}^0B_L , \quad (5)$$

which final decomposition by virtue of nilpotence θ ($\theta^2 = 0$) has

$$X^\mu(\tau, \sigma, \theta) = x^\mu(\tau, \sigma) + i\psi^\mu(\tau, \sigma)\theta , \quad x^\mu \in {}^0B_L , \quad \psi^\mu \in {}^1B_L . \quad (6)$$

We shall identify $x^\mu(\tau, \sigma)$ with coordinates of points of a string in D -dimensional Minkowski space, and $\psi^\mu(\tau, \sigma)$ with D -multiplet majoran fermionic of fields, transforming on vector presentation of Lorentz group $SO(D-1, 1)$.

The group SUSY has linear representation on vector superfields X^μ over $B_L^{2,1}$. The generators of this representation are easily written out:

$$\hat{P}_a = \frac{dT[X]}{d\alpha^a} \Big|_{\alpha=0} = \frac{\partial}{\partial\sigma^a} ; \quad \hat{Q} = \frac{dS[X]}{d\varepsilon} \Big|_{\varepsilon=0} = \frac{\vec{\partial}}{\partial\theta} + i\rho^a\theta \frac{\partial}{\partial\sigma^a} . \quad (7)$$

Covariant derivative operator

$$D = \frac{\vec{\partial}}{\partial\theta} - i\rho^a\theta\partial_a ; \quad \partial_a = \frac{\partial}{\partial\sigma^a} \quad (8)$$

together with generators \hat{P}_a and \hat{Q} derivate algebra SUSY:

$$\begin{aligned} [\hat{P}_a, \hat{P}_b] &= [\hat{P}_a, \hat{Q}] = [D, \hat{P}_a] = 0 ; \\ \{\hat{Q}, D\} &= 0 ; \quad \{\hat{Q}, \hat{Q}\} = -\{D, D\} = 2i\rho^a\hat{P}_a . \end{aligned} \quad (9)$$

The formulas of transformation of components fields x^μ and ψ^μ are easily written out. So at supertransformation S we have

$$\delta x^\mu = -i\varepsilon\psi^\mu ; \quad \delta\psi^\mu = \varepsilon\rho^a\partial_a x^\mu . \quad (10)$$

We shall proceed now to construction of dynamics of a string. We shall posit action, similar to action for bosonic Polyakov's string in conformal gauge [5]

$$S = -\frac{1}{2\pi} \int d^2\sigma \partial_a x^\mu \partial^a x_\mu \quad (11)$$

and passing in action for superparticle Di Vecchia-Ravndal [2]

$$S = \frac{1}{2} \int d\tau \int d\theta \bar{D} X^\mu \bar{D}^2 X_\mu , \quad (12)$$

where $\bar{D} = \frac{\vec{\partial}}{\partial\theta} - i\theta \frac{\partial}{\partial\tau}$.

Minimal action, generalizing (11) for type (12) – it is obvious, that

$$S = -\frac{1}{2\pi} \int d^2\sigma \int d\theta D X^\mu D^2 X_\mu . \quad (13)$$

The lagrangian, adequate such action, can be written out in kind

$$L = -\frac{1}{2\pi} \int d\theta DX^\mu D^2 X_\mu = \frac{1}{2\pi} (\rho^a \rho^b \partial_a x^\mu \partial_b x_\mu + i \rho^a \psi^\mu \partial_a \psi_\mu) . \quad (14)$$

For any action of a type (13)

$$S = \int d^2\sigma L(\partial x, \partial\psi, \psi) \quad (15)$$

from variational principle $\delta S = 0$ equations of a movement can be received

$$\frac{\partial}{\partial\tau} \frac{\partial L}{\partial \dot{x}} + \frac{\partial}{\partial\sigma} \frac{\partial L}{\partial x'} = 0, \quad \frac{\partial L}{\partial\psi} \frac{\partial}{\partial\tau} \frac{\partial L}{\partial \dot{\psi}} - \frac{\partial}{\partial\sigma} \frac{\partial L}{\partial \psi'} = 0, \quad (16)$$

where $\dot{A} \equiv \frac{\partial A^\mu}{\partial\tau}$, $A' \equiv \frac{\partial A^\mu}{\partial\sigma}$ and boundary conditions (for open string)

$$\frac{\partial L}{\partial x'} \delta x = 0; \quad \frac{\partial L}{\partial \psi'} \delta \psi = 0 \quad \text{at } \sigma = 0, \pi. \quad (17)$$

For closed string the boundary conditions (17) are away, instead of them we shall impose a condition of periodicity

$$X^\mu(\tau, 0) = x^\mu(\tau, \pi); \quad \psi^\mu(\tau, 0) = \pm \psi^\mu(\tau, \pi). \quad (18)$$

By virtue of quadric of observable sizes on ψ^μ the conditions (18) do not result in discontinuity of observable values.

Substituting in (16) lagrangian (14), we shall receive equations of a movement

$$\rho^a \rho^b \partial_{ab}^2 x^\mu = 0; \quad \rho^a \partial_a \psi^\mu = 0, \quad (19)$$

which can be rewritten in form

$$\ddot{x} + \alpha^2 x'' + 2\alpha \dot{x}'; \quad \dot{\psi} + \alpha' = 0, \quad (20)$$

where an vectorial index μ is omitted and designation $\alpha \equiv \rho^1/\rho^0$ is entered.

The solution (20) has a kind

$$x = \phi(\sigma - \alpha\tau) + \Phi(\sigma - \alpha\tau)\tau; \quad \psi = \psi(\sigma - \alpha\tau), \quad (21)$$

where ϕ , Φ , ψ – any smooth functions. Boundary conditions (17) bring in requirement

$$\Phi \equiv 0; \quad \psi \equiv 0, \quad (22)$$

that is, despite parabolic type of the first of equations (20), it has the solution as constant wave. The second of conditions (22) means, that the open minimal string can be only bosonic.

For closed string of a condition of periodicity (18) bring in requirement of periodicity of functions ϕ , Φ , ψ on τ with period $\pi\alpha^{-1}$.

For finding-out of physical sense of the solution (21) we shall find canonical density of a momentum for (14)

$$p^a = -\frac{\partial L}{\partial \partial_a x} = -\frac{1}{\pi} \rho^a \rho^b \partial_b x. \quad (23)$$

Substituting here decision (21), we shall receive

$$p^0 = -\frac{1}{\pi} (\rho^0)^2 \Phi, \quad p^1 = -\frac{1}{\pi} \rho^0 \rho^1 \Phi. \quad (24)$$

The momentum of the whole string is received by integration

$$P = \int_0^\pi p^0 d\sigma = -\frac{1}{\pi} (\rho^0)^2 \int_0^\pi \Phi(\sigma - \alpha\tau) d\sigma. \quad (25)$$

Hence, for open string $P = 0$. For closed string we shall choose the even solution $\Phi = -(\rho^0)^{-2} P = \text{const}$, which answers a movement of a string as whole with momentum P . Then the solution (21) can be rewritten in kind

$$X^\mu(\tau, \sigma) = x^\mu(0) + P^\mu \tau + \sum_{n \neq 0} \frac{a_n^\mu}{n} e^{-2in(\alpha\tau\sigma)}. \quad (26)$$

Is here restored D -dimensional an index μ and the function $\phi^\mu(\sigma - \alpha\tau)$ is spreaded out on waves, running left to right at $\alpha > 0$ (R -mode) and to the left at $\alpha < 0$ (L -mode).

The fermionic sector of a closed string as against open non-trivial and consists of four parts: L - R , L - NS , R - R , R - NS . The second symbol means a type of periodic conditions (18)

$$\begin{aligned} \psi(\tau, 0) &= \psi(\tau, \pi) - \text{Ramond } (R), \\ \psi(\tau, 0) &= -\psi(\tau, \pi) - \text{Neveu - Schwarz } (NS), \\ \psi^\mu(\tau, \sigma) &= \sum_{n \in K} b_n e^{-2in(\alpha\tau\sigma)}, \\ K &= \begin{cases} Z & \text{Ramond } L - R (\alpha < 0) \text{ and } R - R (\alpha > 0) \\ Z + \frac{1}{2} & \text{Neveu - Schwarz } L - NS (\alpha < 0) \text{ and } R - NS (\alpha > 0). \end{cases} \end{aligned} \quad (27)$$

For research of constructed model we shall find Noether currents [1].

From Lorentz-invariancy of lagrangian (14) law of preservation follows

$$\partial_a J_{\mu\nu}^a = 0, \quad (28)$$

Where $J_{\mu\nu}^a = -\frac{\rho_a}{\pi} [\rho^b (x^\mu \partial_b x^\nu - x^\nu \partial_b x^\mu) + i\psi^\mu \psi^\nu]$ - density of tensor of a angular momentum. This law provides preservation of a angular momentum of a string, calculated on any spatial-like curve, crossing an once a world surface of a string.

Noether currents, caused by invariancy (14) concerning Poincaré-translations - this bosonic p_μ^a and fermionic π_μ^a of density of a momentum

$$p_\mu^a = -\frac{1}{\pi} \rho^a \rho^b \partial_b x_\mu; \quad \pi_\mu^a = \frac{i}{2\pi} \rho^a \psi_\mu. \quad (29)$$

The laws of preservation - it is simple equations of a movement (19).

By virtue of invariancy (14) concerning sheet translations T (1) we have the law of preservation of tensor of an energy-momentum

$$\begin{aligned} \partial_b T_b^a &= 0, \\ 2\pi T_b^a &= 2\rho^a \rho^c \partial_c x^\mu \partial_b x_\mu + i\rho^a \psi^\mu \partial_b \psi_\mu - \delta_b^a (\rho^c \rho^d \partial_c x^\mu \partial_d x_\mu + i\rho^c \psi^\mu \partial_c \psi_\mu). \end{aligned} \quad (30)$$

And, at last, by virtue of invariancy (14) concerning supertransformation of components fields (10) we have a preserved supercurrent J^a

$$\partial_a J^a = 0, \quad 2\pi J^a = -i\rho^a \rho^b \partial_b x^\mu \psi_\mu. \quad (31)$$

As well as at all string models in theory of a minimal string there are the constraints. So from (29) follows, that bosonic and fermionic density of a momentum of a string submit to constraints

$$P_1^\mu = \alpha P_0^\mu, \quad \pi_1^\mu = \alpha \pi_0^\mu. \quad (32)$$

The choice α can be concretized, if to notice, that $\rho^a \rho_a = (\rho^0)^2 (\alpha^2 - 1)$. Assuming $\rho^a \rho_a \equiv 0$, we have $\alpha = \pm 1$, and, hence, the components of a supercurrent will satisfy in this case to an additional constraints

$$\rho^a J_a = 0. \quad (33)$$

Easily to see, that by virtue of equations of a movement (20) the tensor of an energy-moment is traceless, as well as in bosonic case at $N = 0$

$$Tr T_{ab} = 0. \quad (34)$$

Counting T_{ab} symmetric, we shall receive a condition

$$(\dot{x} + \alpha x')^2 = P^\mu P_\mu = 0, \quad (35)$$

that is string as whole moves with velocity of light.

By analogy to coordinates of a light cone in purely bosonic case we shall enter semicone coordinates

$$t = \tau, \quad \xi = \sigma - \alpha \tau. \quad (36)$$

Then in these coordinates tensor of an energy-momentum of signs a kind

$$2\pi T_{ab} = \begin{pmatrix} T_{tt} & T_{t\xi} \\ T_{\xi t} & T_{\xi\xi} \end{pmatrix} = -(\rho^0)^2 \begin{pmatrix} -2(\dot{x} + \alpha x')^2 & (\dot{x} + \alpha x')^2 \\ (\dot{x} + \alpha x')^2 & \dot{x}^2 - x'^2 + \frac{i}{\rho^0} \psi \dot{\psi} \end{pmatrix}, \quad (37)$$

and vector of a supercurrent will change

$$J_t = 0, \quad J_\xi = -\frac{i}{2\pi} \rho^0 \rho^1 (\dot{x} + \alpha x') \psi. \quad (38)$$

The constraints then can be compactly written in form

$$T_{tt} = 0, \quad T_{\xi\xi} = 0, \quad J_\xi = 0 \quad (39)$$

or in components

$$(\dot{x} + \alpha x')^2 = 0; \quad \dot{x}^2 - x'^2 + \frac{i}{\rho^0} \psi \dot{\psi} = 0; \quad (\dot{x} + \alpha x') \psi = 0. \quad (40)$$

The condition (39) or (40) can be posited as Virasoro superconditions. It is desirable, certainly, to have a conclusion of these conditions from more regular procedure, connected to fixing of calibration in lagrangian of a string, constructed for type of a 2-dimensional supergravity. Such problem is much difficult and represents the following stage of development of the theory. The Virasoro conditions are more important for construction of the quantum theory of a minimal string, that is problem, which is not mentioned in given work.

From constructed classical theory of a minimal string it is visible, that the received design is excellent from Neveu-Schwarz-Ramond string: asymmetry right and left-hand; non-triviality fermionic sectors only for closed strings; a movement of the whole string as whole with velocity of light; a parabolic type of equations of a movement and e.t.. As the theory of a minimal string in some sense is more simple, than the standard theory of Neveu-Schwarz-Ramond, are necessary further researches of this model, especially in quantum area.

References

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